Hadronic Spectral Functions in Lattice QCD at Finite Tepmerature

T. Hatsuda (Univ. of Tokyo)

Contents

CP-PACS !

MELQCD Collaboration

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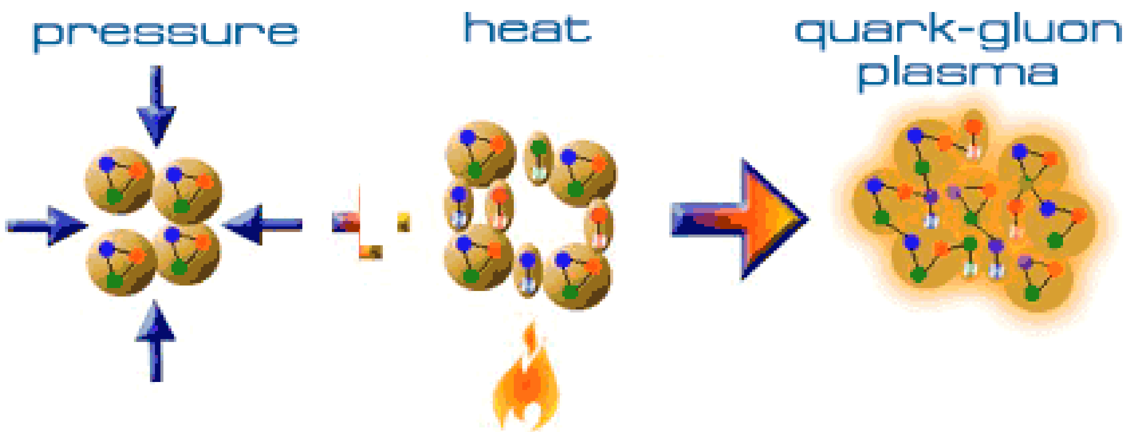
- Asakawa, Nakahara + T.H, Phys. Rev. D60, 091503 ('99)
  Prog. Part. Nucl. Phys. 46, 459 ('01)
- Asakawa, Nakahara + T.H., hep-lat/0208059 ('02)
- Sasaki, Sasaki, Asakawa +T.H, hep-lat/0209059 ('02)

SR2201
(JAERI)

CP-PACS

SR8000(KEK)

高温・高密度における新物質相

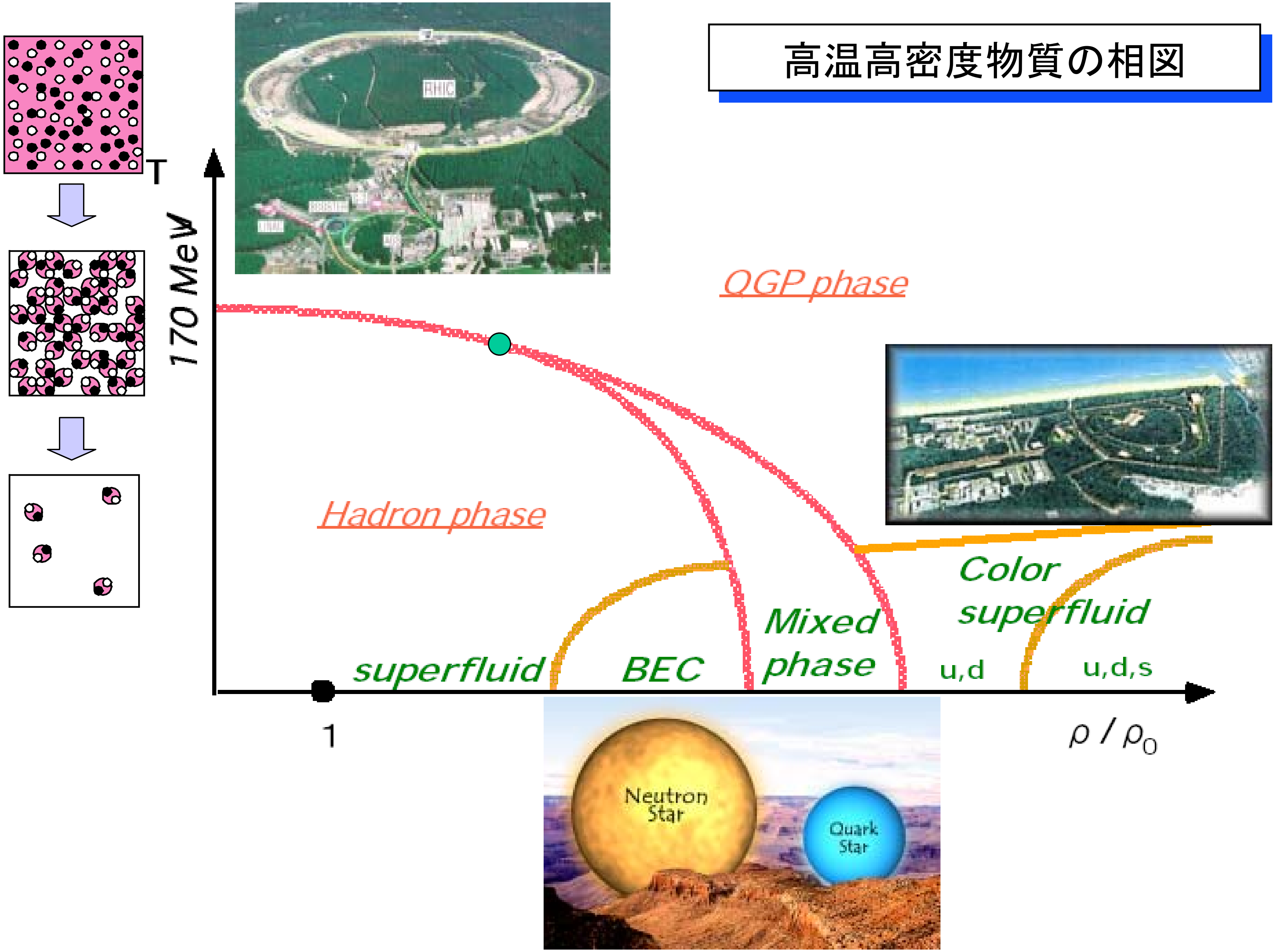


臨界密度: 10^{12} kg/cm^3　　*臨界温度: 10^{12} K*

どこで実現？

- *初期宇宙* ($t < 10^{-4}$ sec) *:*　*高温・低バリオン密度*
- *中性子星中心部:*　*低温・高バリオン密度*
- *相対論的重イオン衝突:*　*高温* and/or *高バリオン密度*

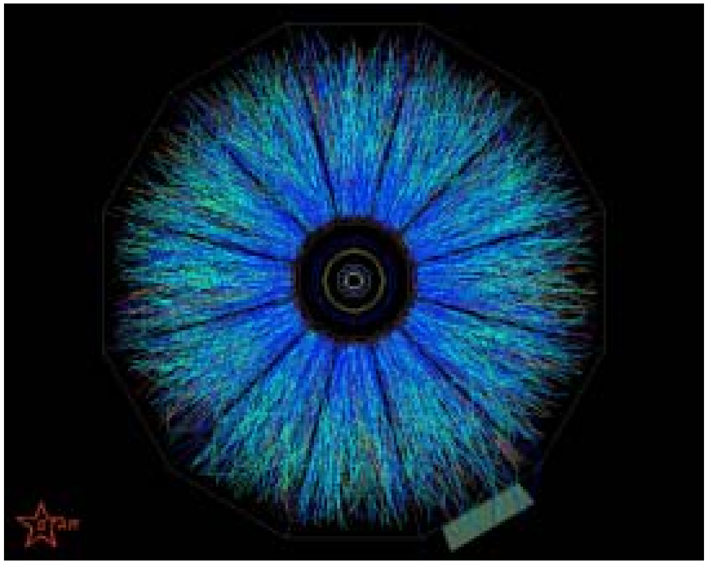
高温高密度物質の相図

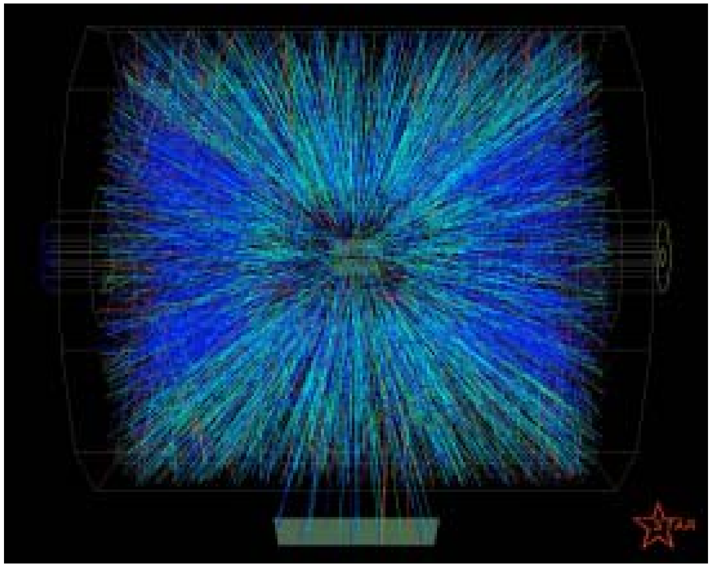


T
170 MeV
QGP phase
Hadron phase
superfluid
BEC
Mixed phase
Color superfluid
u,d
u,d,s
1
ρ / ρ_0
RHIC
Neutron Star
Quark Star

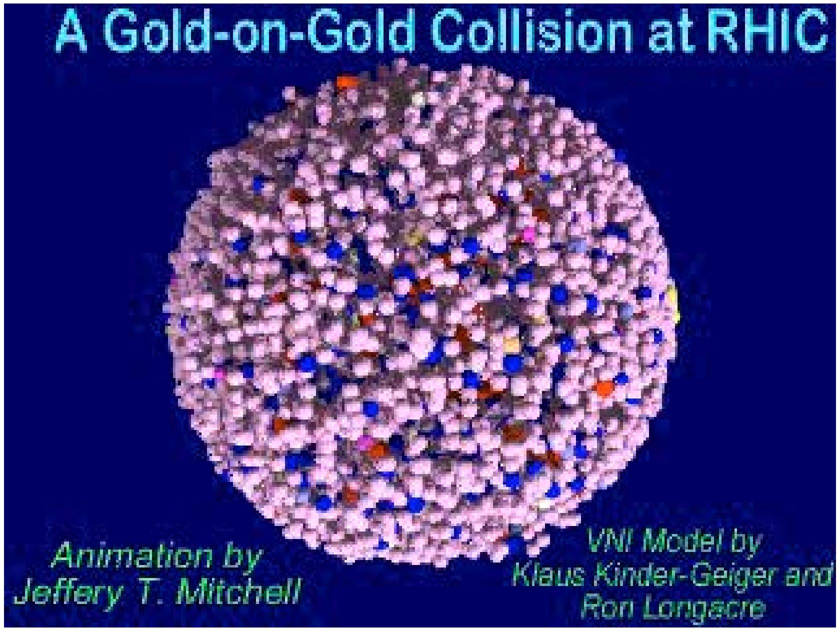
実験室におけるQGP探索

実験 (RHIC)





数値シミュレーション
(パートンカスケード)



量子色力学

$$L = -\frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \bar{q}\gamma^{\mu}(i\partial_{\mu} - g\, t^{a} A^{a}_{\mu})q - m\bar{q}q$$

南部陽一郎 (1966)

Discretization

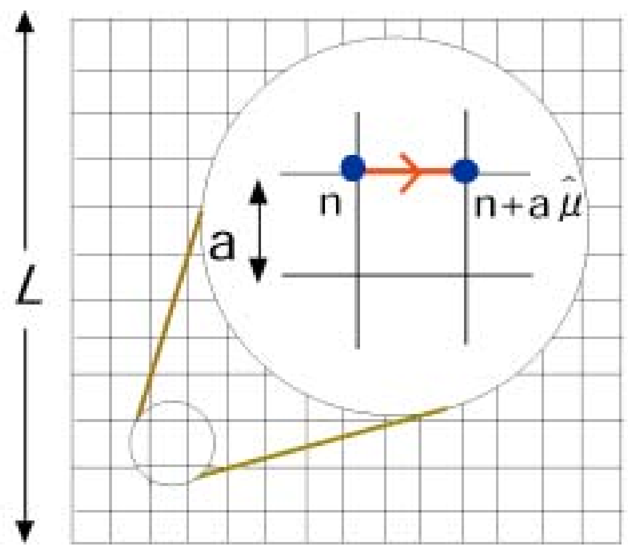
$A_{\mu}(x) \rightarrow U_{\mu}(n) = \exp(i\, a\, A_{\mu})$

$q(x) \rightarrow q(n)$

$T=0 : L^3 \times L = (N_s a)^4$

$T \neq 0 : L^3 \times 1/T = (N_s a)^3 \times (N_t a)$

4-d Euclidean lattice

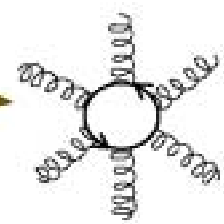


K. Wilson (1973)

Monte Carlo integration

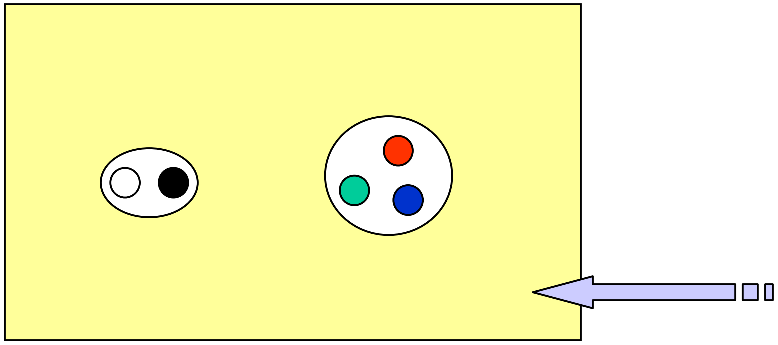
$$Z = \int [dU][dq d\bar{q}]\ e^{-S_{Dirac}(q,\bar{q},U) - S_{YM}(U)}$$

$$= \int [dU]\ \det Q(U)\ e^{-S_{YM}(U)}$$



- *Quenched QCD :* $\det Q \rightarrow 1$
- *Full QCD :* $\det Q \neq 1$

Hadrons in Hot Environment

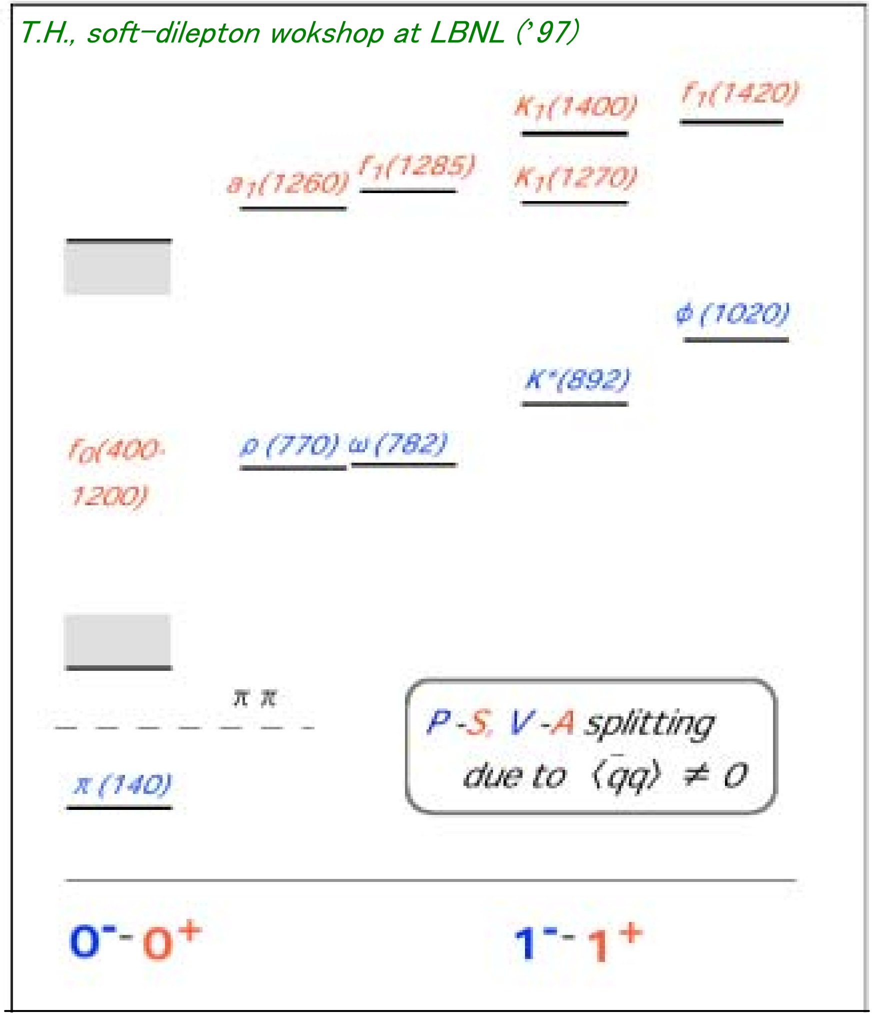


$\langle \bar{q}q \rangle \neq 0$

● *軽いクォーク系とカイラル対称性:* *Hatsuda & Kunihiro, PRL ('85)*

● *重いクォーク系と閉じ込め:* *Matsui & Satz, PLB ('86)*
Miyamura et al., PRL ('86)

Chiral partners in the vacuum



Tracer of the chiral structure of matter

$\langle \bar{q} \bullet q(x) \; \bar{q} \bullet q(y) \rangle$

- $\langle \bar{q}q \rangle \to 0$

  ⇔ Chiral degeneracy

  $\langle S(x)S(y) \rangle \sim \langle P(x)P(y) \rangle$

  $\langle A(x)A(y) \rangle \sim \langle V(x)V(y) \rangle$

- change of $\langle \bar{q}q \rangle$

  ⇔ Individual spectrum

  $\langle S(x)S(y) \rangle , \langle P(x)P(y) \rangle$

  $\langle A(x)A(y) , \langle V(x)V(y) \rangle$

First principle QCD calculation ?

QCD Spectral Functions

Lattice data

$$D(\tau,\vec{p}) = \int \left\langle J^{+}(\tau,\vec{x}) J(0,0) \right\rangle e^{i\vec{p}\vec{x}} d^3x$$

$$= \int K(\tau,\omega)\, A(\omega,\vec{p})\, d\omega$$

"Laplace" kernel

$K(\tau,\omega)$

$= e^{-\omega\tau}/(1 \mp e^{-\omega/T})$

Spectral Function

All information on hadronic correlations
at T=0 and T≠0

pQCD says …

* $A(\omega \geq 0) = \mp A(-\omega) \geq 0$

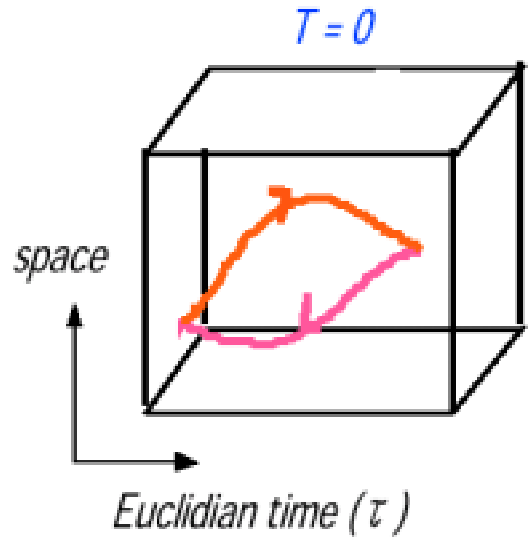
* $A(\omega >> 1\,\mathrm{GeV}) \rightarrow \omega^{2\dim[O]-4}\left(1 + c\frac{\alpha_s}{\pi}\right)$

How to extract A(ω) from lattice QCD data?

$$D(\tau) = \int \langle \bar{q}\Gamma q(\tau,\vec{x})\, \bar{q}\Gamma q(0) \rangle\, d^3x$$

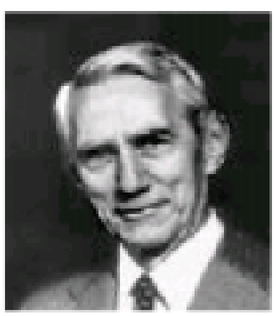
$$\sim e^{-m\tau} \qquad (\tau \to \infty)$$

$$= \int \frac{e^{-\omega\tau}}{1 \mp e^{-\omega/T}}\, A(\omega)\, d\omega \qquad (\, any\ \tau\,)$$



Ill-posed problem

T. Bayes 1702-1761



C.E. Shannon, 1916-2001

Maximal Entropy Method : $P[A \mid D]$

- No parametrization of $A(\omega)$
- unique solution for $D(\tau) \to A(\omega)$
- error estimate on $A(\omega)$

Reviews :

Optics and astrophysics: N. Wu, Springer ('97)

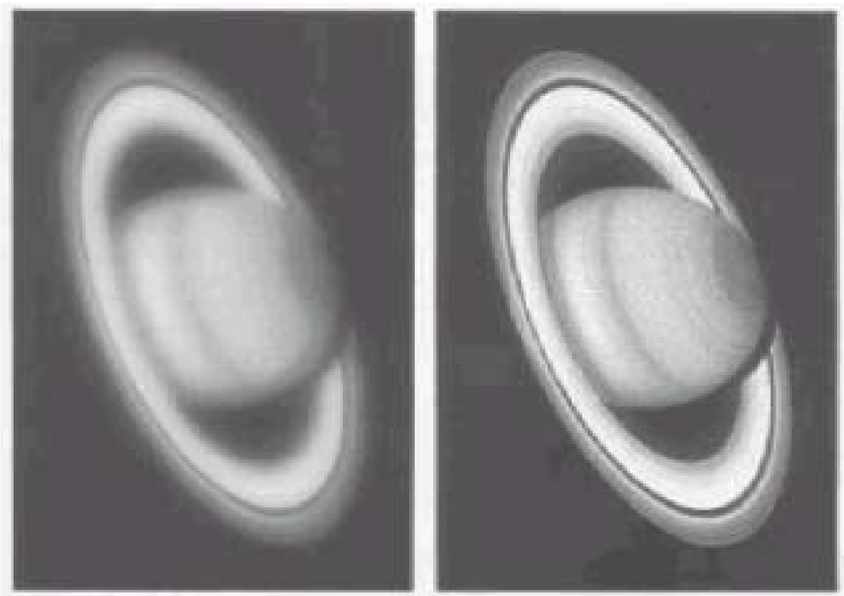
Spin systems: Jarrell & Gubernatis, Phys. Rep. 269 ('96)

Lattice QCD: Asakawa, Nakahara and T.H., Prog. Part. Nucl. Phys. 47 ('01)

MEM Image Reconstruction

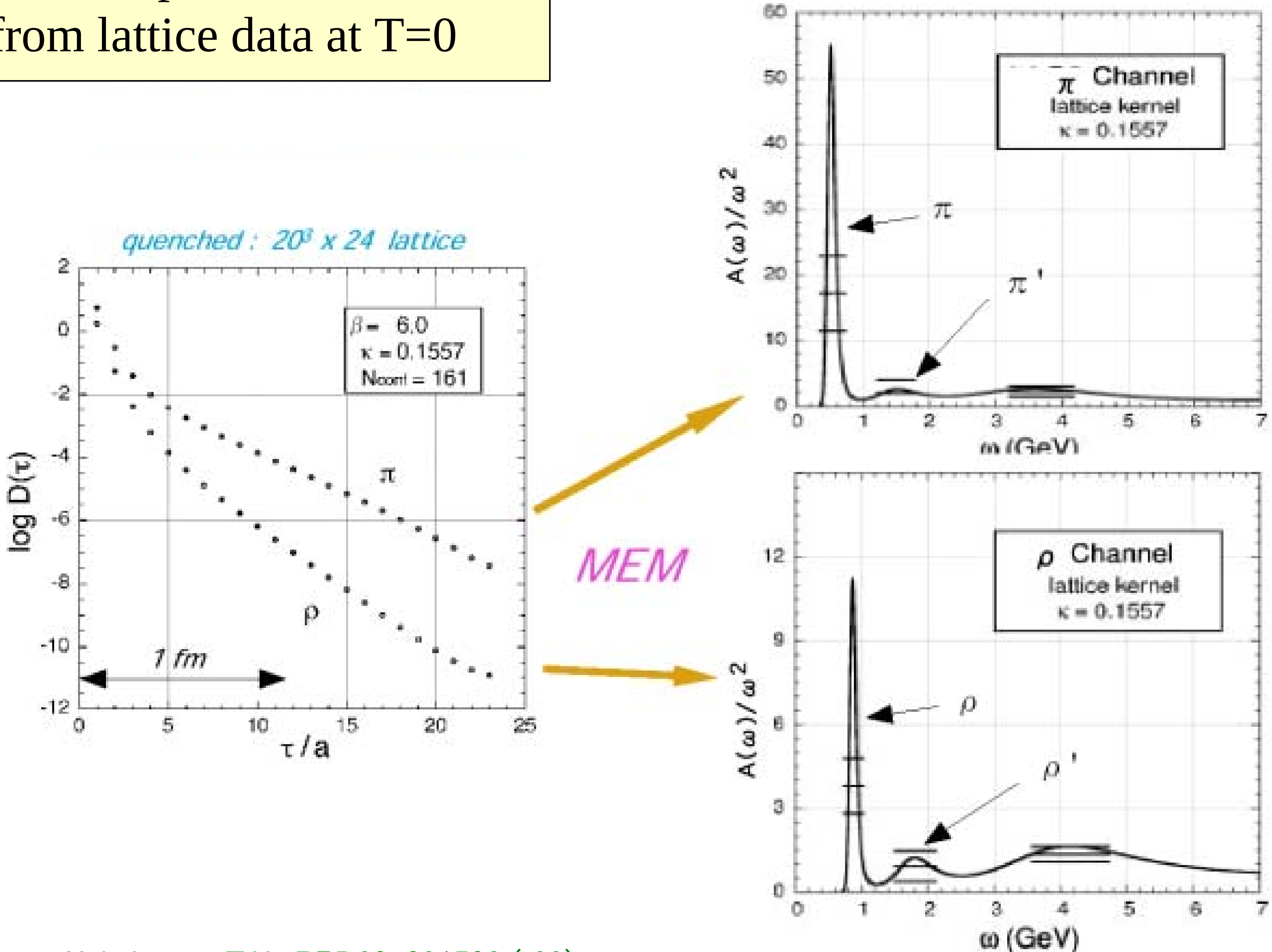
The girl's portrait

The Image of Saturn



From N. Wu, "The Maximum Entropy Method" (1997)

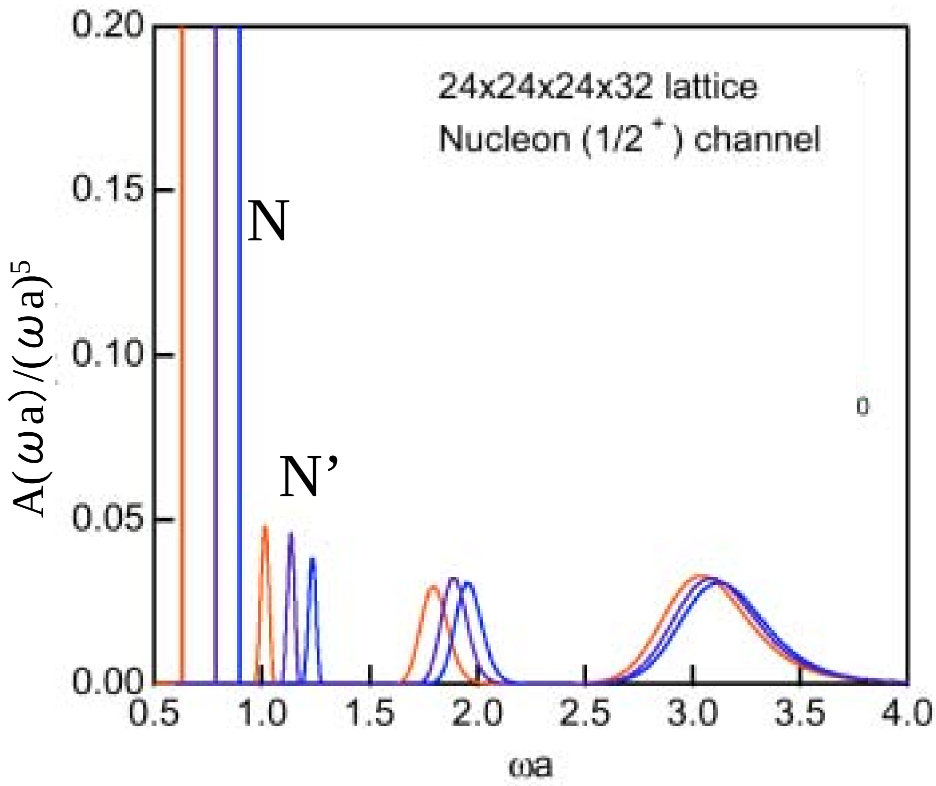
π & ρ spectral functions
from lattice data at T=0

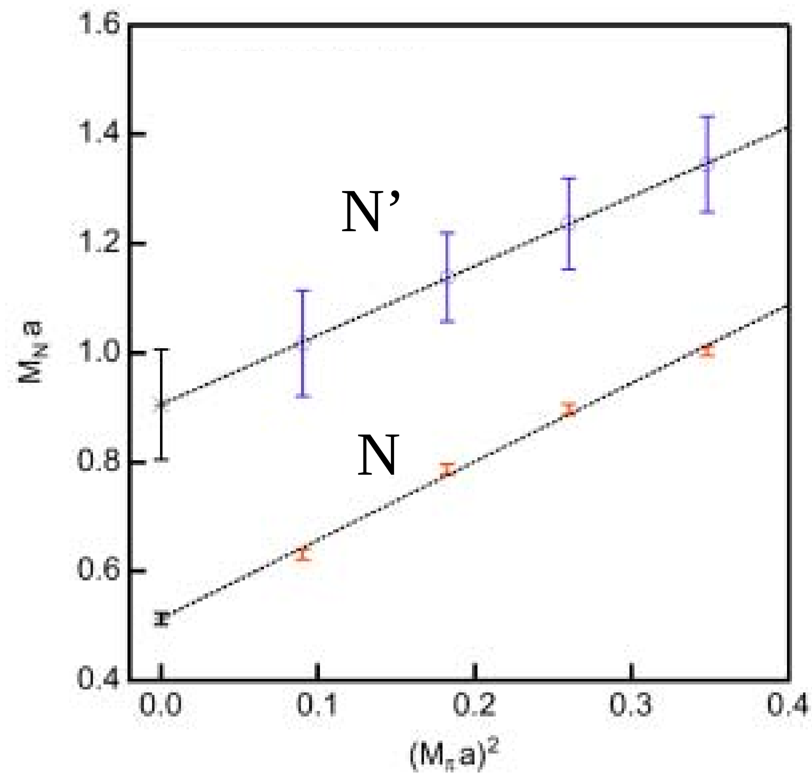


Asakawa, Nakahara + T.H., PRD60, 091503 ('99)

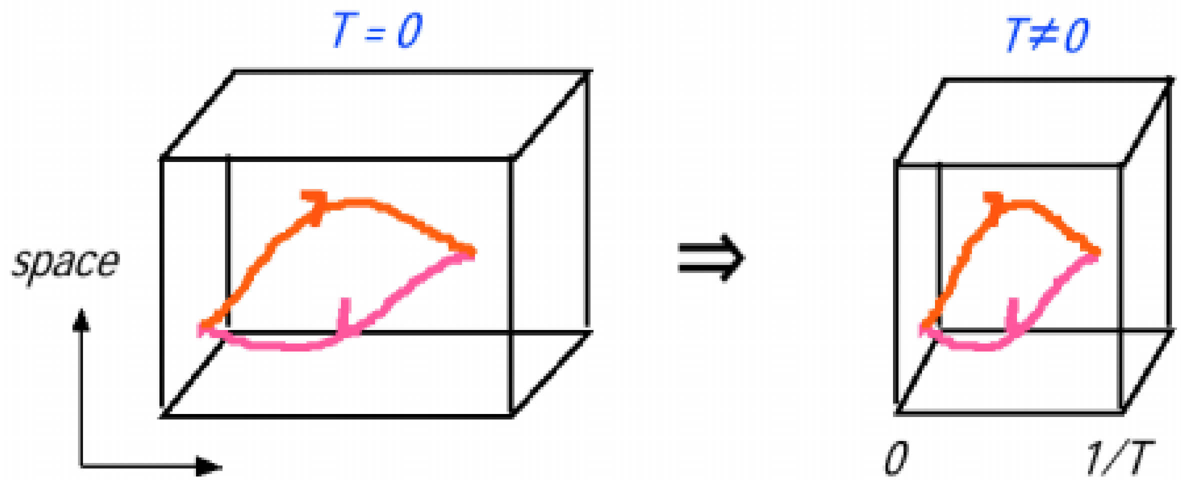
Nucleon spectral function from lattice data at T=0

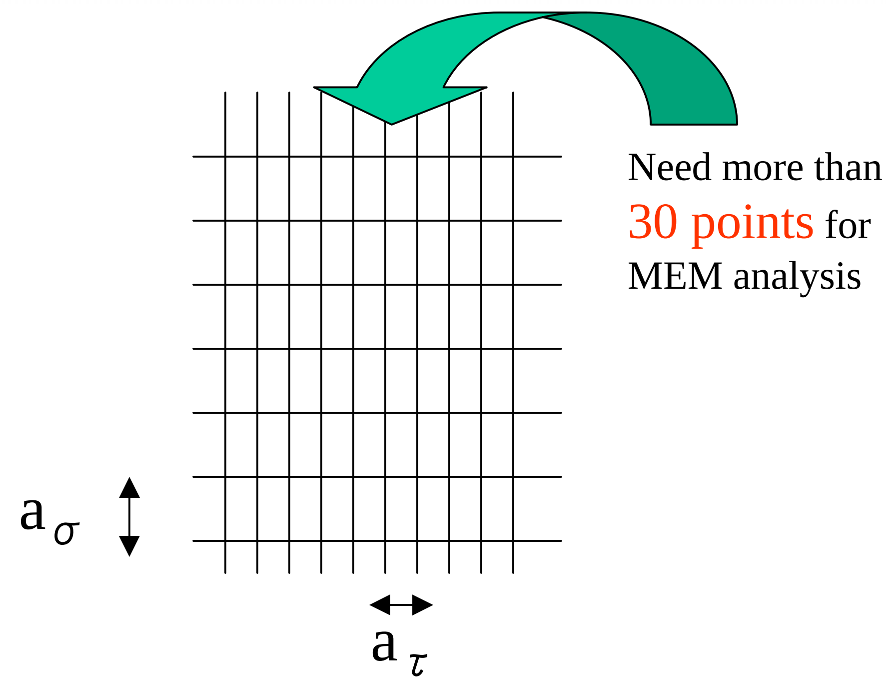
Sasaki, Sasaki, Asakawa + T.H., hep-lat/0209059 ('02)





Need of aniostropic lattice at T≠0





Anisotropic lattice

β =7.0, $32^3 \times N_\tau$

$a_\tau = a_\sigma/4 \approx$ 0.01 fm

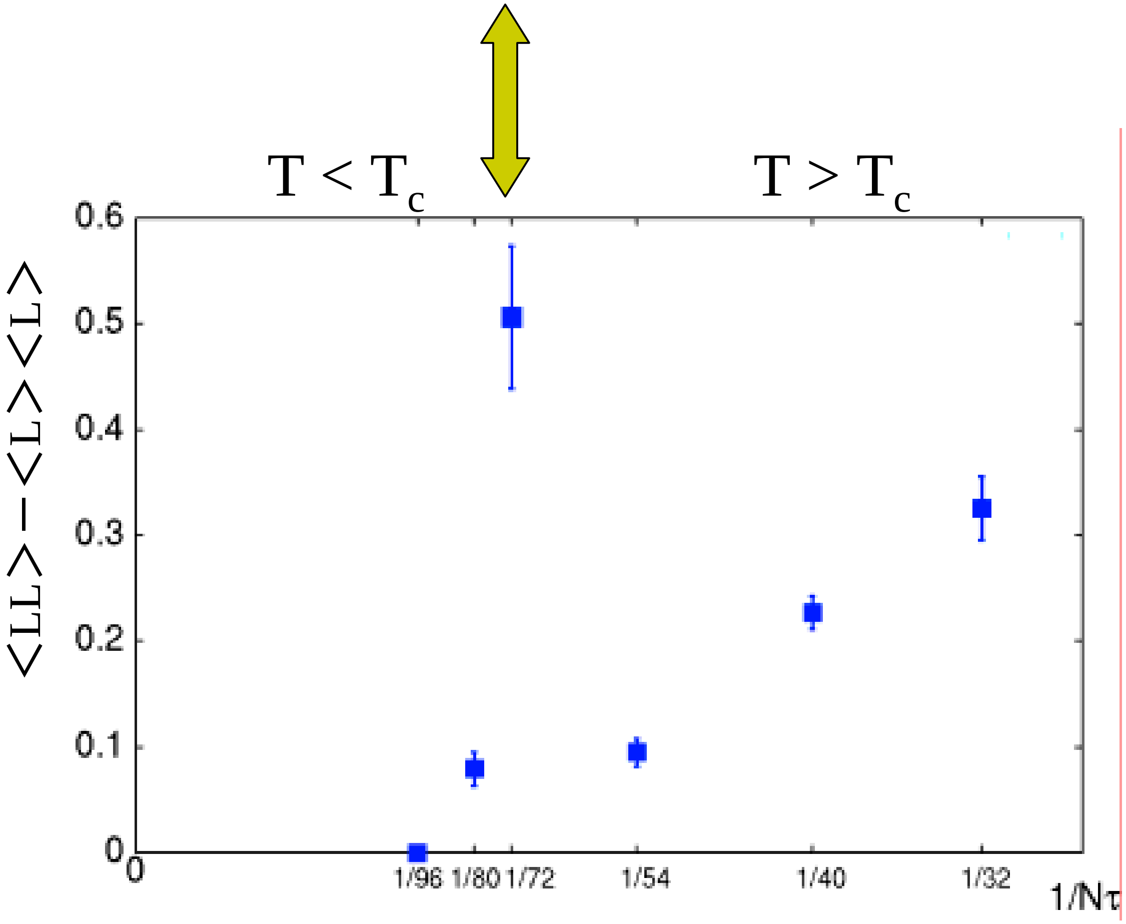
quenched approx.

Machine: CP-PACS

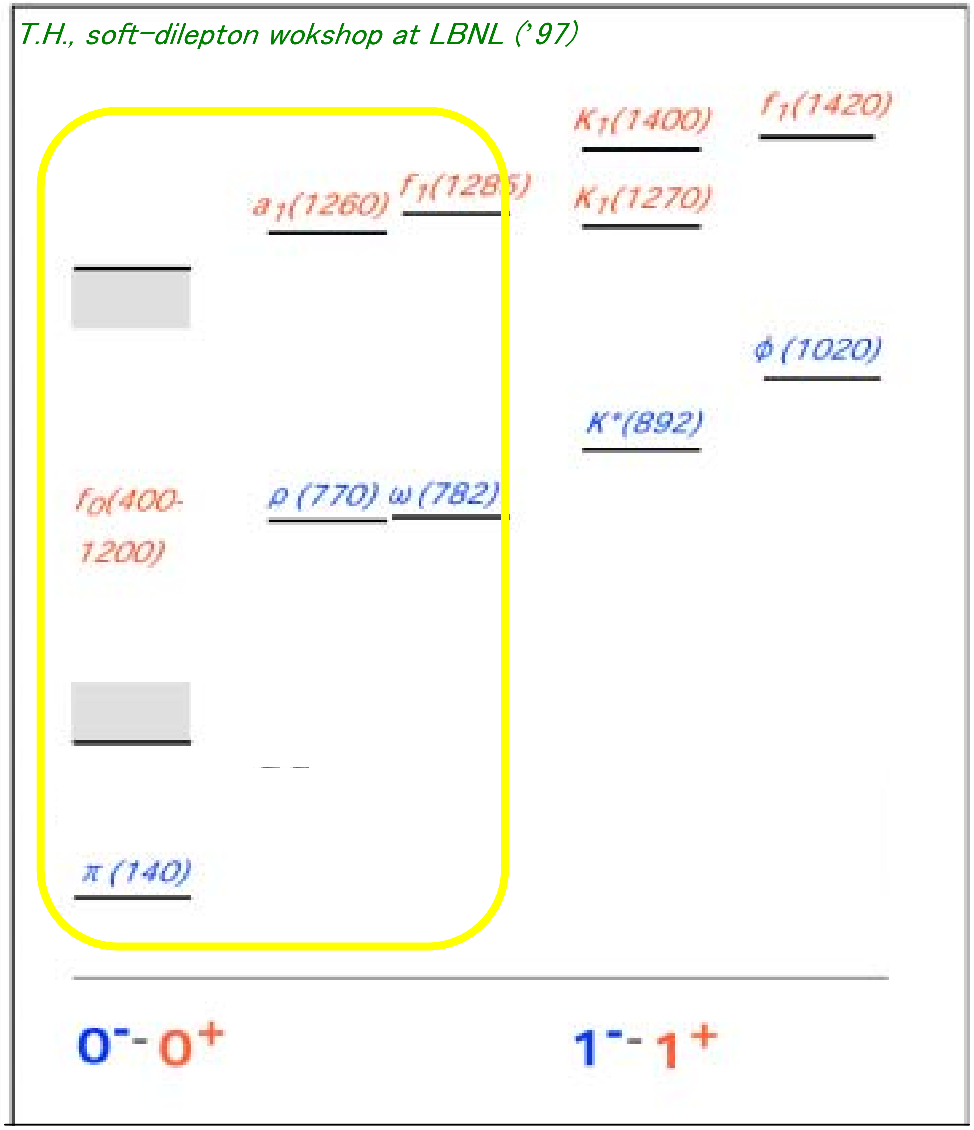
Asakawa & T.H., hep-lat/0208059 ('02)

Temporal lattice size and T

N_τ	96	80	72	54	46	40	32
T / T_c	0.78	0.93	1.04	1.4	1.6	1.9	2.3

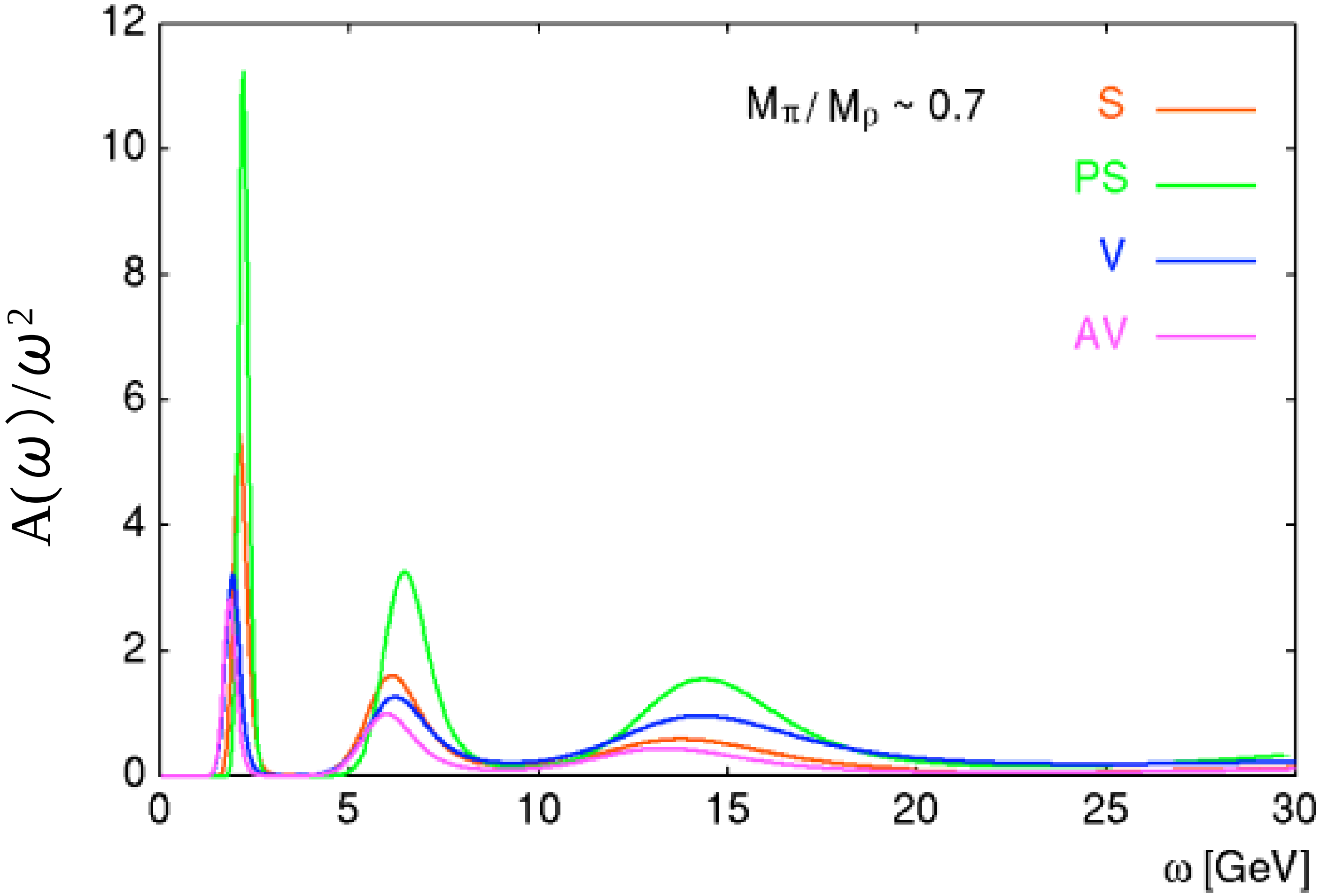


Chiral partners in the vacuum



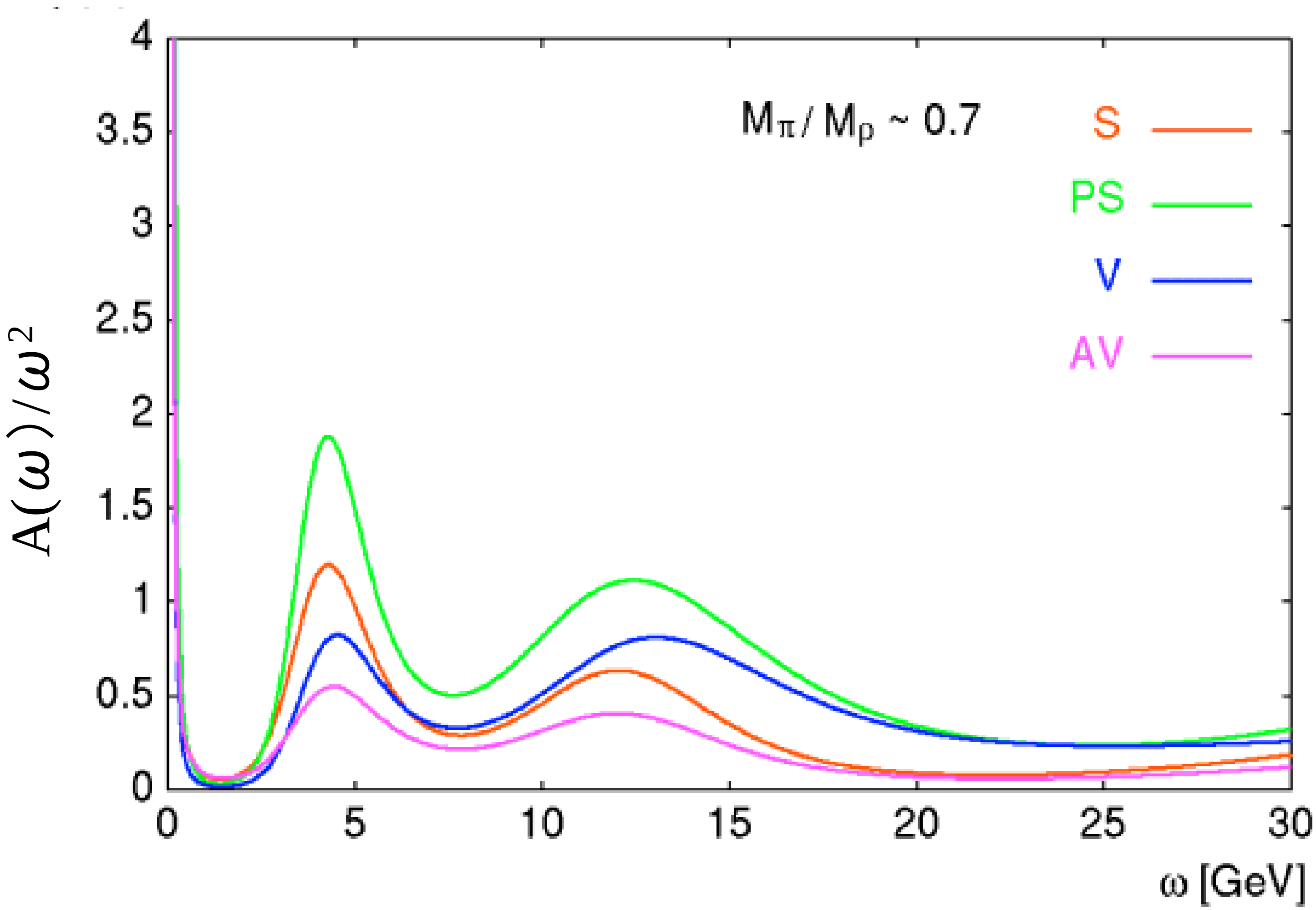
Chiral degeneracy and non-trivial modes ($T \gtrsim T_c$)

$T = 1.4\ T_c$



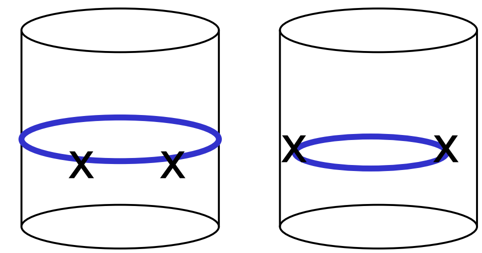
Disappearance of non-trivial modes ($T \gtrsim 2T_c$)

$T = 1.9\ T_c$

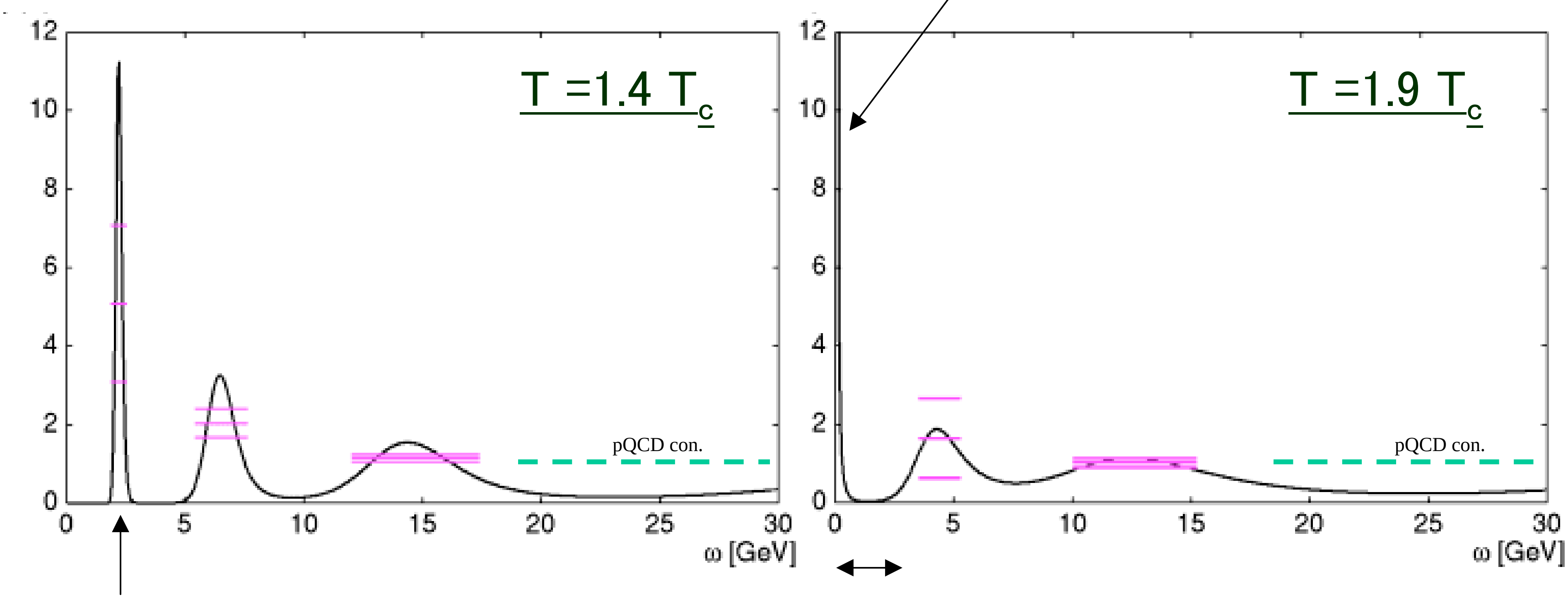


Comparison of T = 1.4 T_c and 1.9 T_c

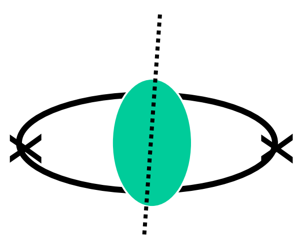
PS Channel $(M_\pi / M_\rho = 0.7)$



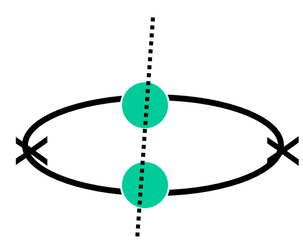
Inverse Cherenkov effect？
Koike, Lee + T.H., NPB ('93)



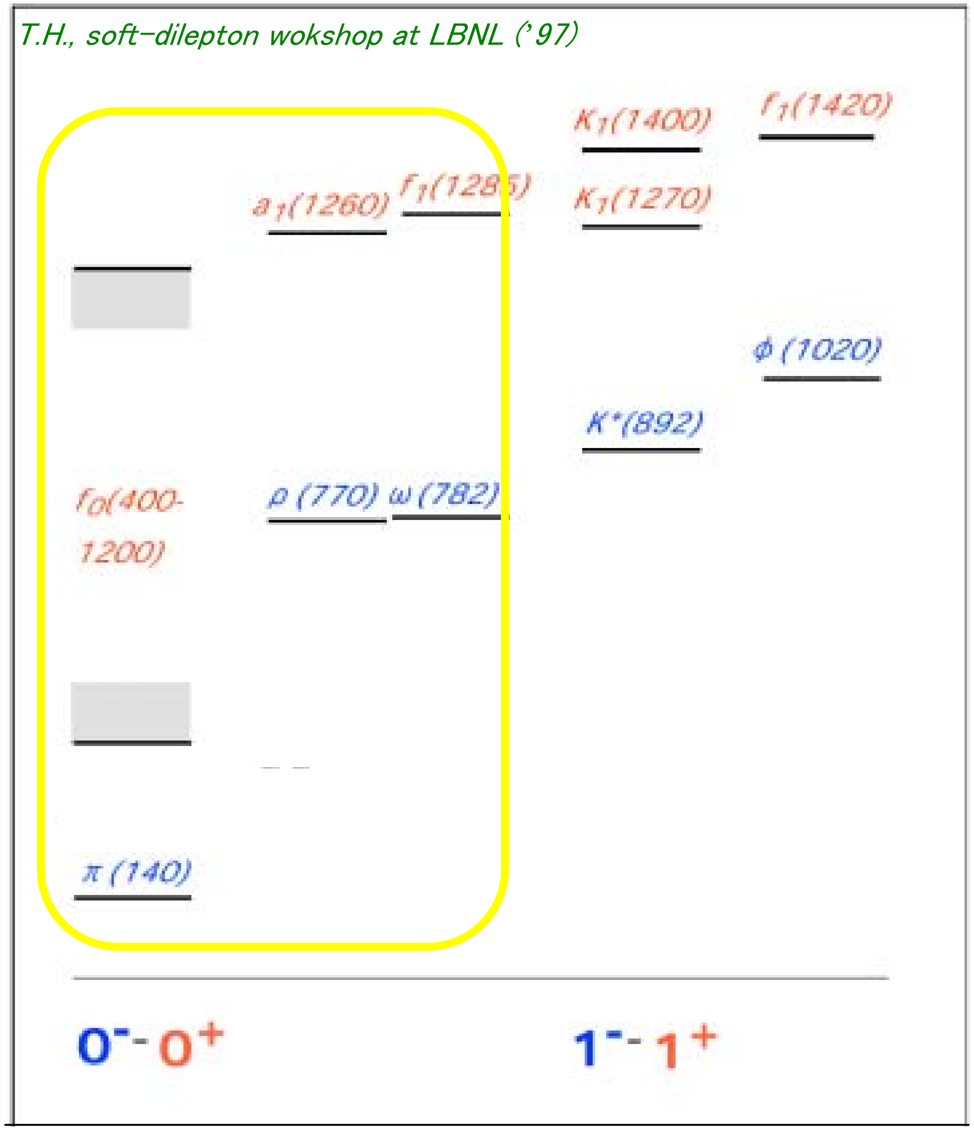
Para-pion mode？
Kunihiro + T.H., PRL ('85)

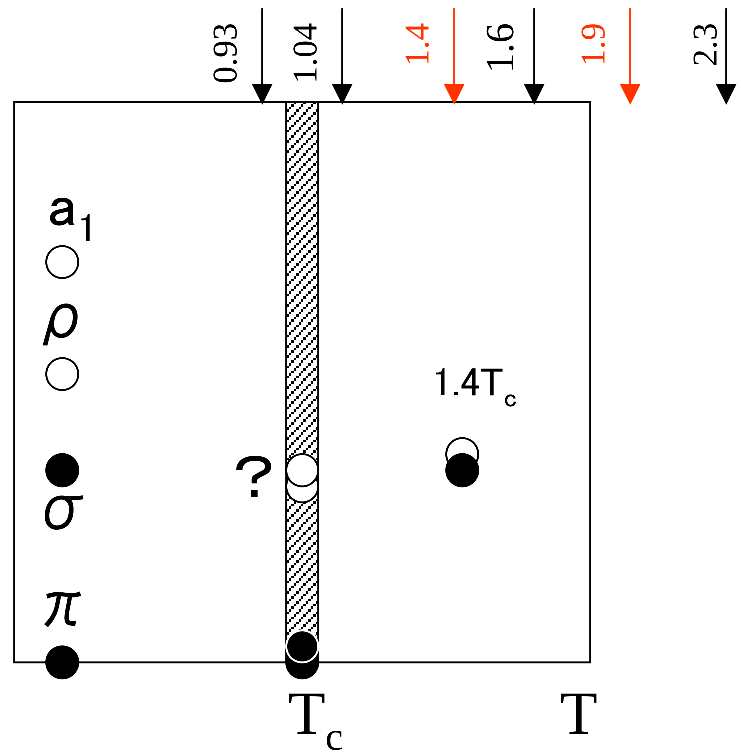


Plasmino gap？
Weldon, PRD ('82)



Hadronic modes below and above the chiral transition





Spectral function (temporal correlation):
Asakawa & T.H., hep-lat/0209059 ('02)

Summary

[1] MEM : new way of analyzing lattice QCD data

- $D(\tau) \rightarrow A(\omega)$: parameterization free, unique solution, significance test
- better lattice data → better $A(\omega)$ Asakawa, Nakahara+T.H., (MELQCD, Tokyo)

[2] T=0 : ground and excited states

- useful for hadron spectroscopy (π^*, ρ^*, N*, Δ^*, glueballs etc)

Sasaki+Sasaki., (MELQCD, Tokyo)

[3] T≠0 : spectral change of hadrons (thanks to CP-PACS !)

- non-trivial collective modes exist up to $2T_c$?
- evidence of the plasmino gap ?

Asakawa +T.H., (MELQCD, Tokyo)

[4] T≠0 : future

- ρ, ω, ϕ vs chiral restoration, J/ψ, ψ' vs deconfinement
- transport coefficients in hot plasma
- collective modes in N_c=2 dense QCD

(拡大MELQCD)

Parameters

1. Lattice size

$$32^3 \times 32 \quad (T \simeq 2.5\,T_c)$$
$$40 \quad (T \simeq 2.0\,T_c)$$
$$54 \quad (T \simeq 1.5\,T_c)$$
$$72 \quad (T \simeq 1.1\,T_c)$$
$$80 \quad (T \simeq 1.0\,T_c)$$
$$96 \quad (T < T_c)$$

2. $\beta = 7.0, \quad \xi_0 = 3.5$

3. $\xi = a_\sigma / a_\tau = 4$

   $a_\tau = 9.75 \times 10^{-3}$ fm

   $L_\sigma = 1.25$ fm

4. Naive Unimproved Action

5. Wilson Fermion

6. Heatbath : Overrelaxation
   = 1 : 4

   1000 sweeps between measurements

7. Four Quark Masses

   $m_\pi / m_\rho \simeq 0.7, 0.8, 0.9$

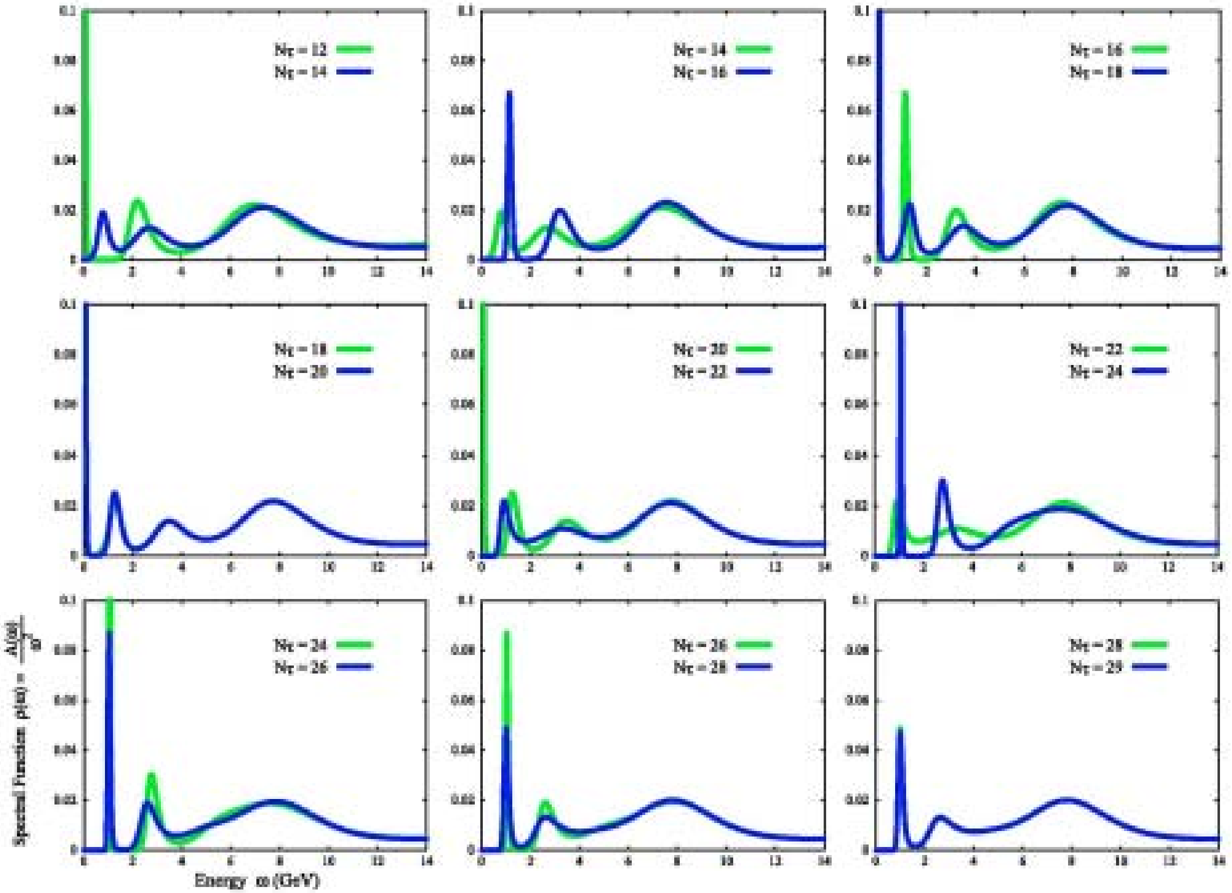
   $m_V = m_{J/\psi}$

8. Quenched Approximation

9. Gauge Unfixed

10. $\vec{p} = \vec{0}$ Projection

11. Machine CP-PACS



$40^3 \times 30$ lattice

$\beta = 6.47$

isotropic lattice

$N_\tau \simeq 30$ or larger : needed

Principles of MEM

$D(\tau) \rightarrow A(\omega)$: *not unique !*

What is the most probable $A(\omega)$ for given $D(\tau)$?

Statistical Inference

Posterior prob. ↓ *likelihood func.* ↓ *Prior prob.* ↓

$$P[A|D] = \frac{1}{P[D]} P[D|A]\, P[A]$$

- $\frac{\delta P[A|D]}{\delta A} = 0 \Rightarrow A_{out}(\omega)$
- $\frac{\delta^2 P[A|D]}{\delta A \delta A} \Rightarrow$ reliability of $A_{out}(\omega)$

T. Bayes 1702-1761

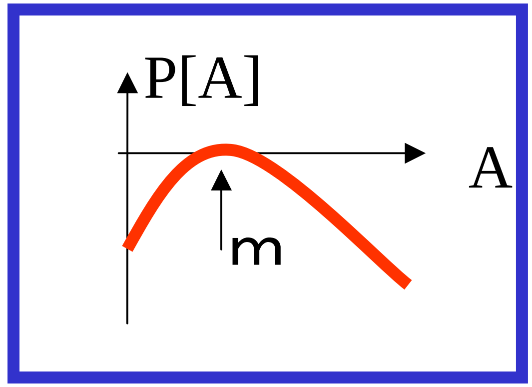
Bayes Theorem

$$P[X\,Y] = P[X|Y]\, P[Y] = P[Y|X]\, P[X]$$

$P[D|A]$: Normal distribution (central limit theorem)

$P[A]$: Shanon-Jaynes information entropy

► *Explicit form of the posterior prob.*



$$P[A|D] \propto P[D|A]\ P[A] \propto e^{Q(D,A)}$$

- *Central limiting theorem*

$$P[D|A] \propto e^{-L(D,A)} = \exp\left(-\frac{1}{2}\sum_{i,j}\left[D(\tau_i) - D_A(\tau_i)\right] C_{ij}^{-1}\left[D(\tau_j) - D_A(\tau_j)\right]\right)$$

- *Information entropy (Shanon-Jaynes)*

$$P[A] \propto e^{\alpha S(A)} = \exp\alpha\int_0^{\infty}\left[A(\omega) - m(\omega) - A(\omega)\ln\left(\frac{A(\omega)}{m(\omega)}\right)\right]d\omega$$

- *Q(D,A) = α S - L : "- free energy"* ⇦ *<u>to be maximized</u> with respect to A(ω)*
- *α : a fictitious "temperature"* ⇦ *<u>to be integrated</u> with a weight P[α|D]*
- *m(ω) : prior estimate of A(ω)* ⇦ *<u>to be updated</u> by error analysis*

▸ *Procedure of modern MEM*

Step 1 *Maximizing Q*

$$\frac{\delta Q}{\delta A(\omega)} = 0 \longrightarrow A_\alpha(\omega)$$

- *Solution is unique (Asakawa, T.H., Nakahara ('00))*
- *Rapid convergence by SVD (Bryan, EBJ ('90))*

Step 2 *averaging over α*

$$A_{out}(\omega) = \int A_\alpha(\omega)\, P[\alpha \,|\, D]\, d\alpha$$

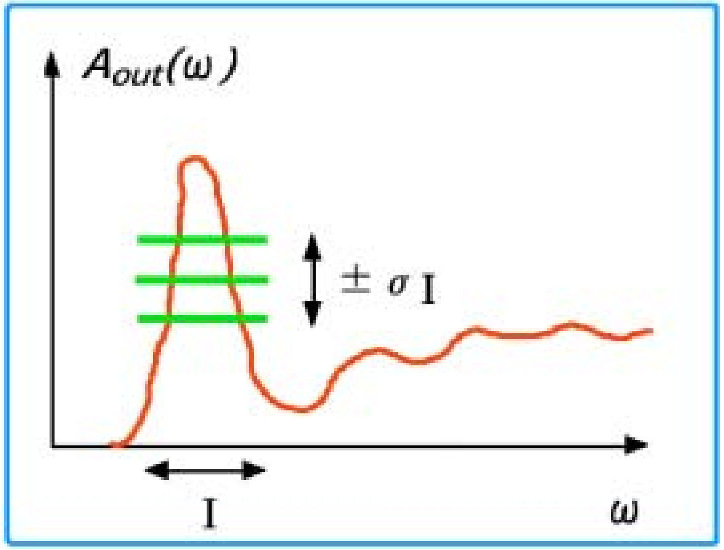
with

$$P[\alpha \,|\, D] \sim \int [dA]\, P[D|A\alpha]\, P[A|\,\alpha]\, P[\alpha]$$

$$\ln P[\alpha \,|\, D] = const. + \frac{1}{2}\sum_k \ln \frac{\alpha}{\alpha + \lambda_k} + Q(D, A_\alpha)$$

Step 3 *error analysis*

$$\sigma_I^2 = -\left\langle \left(\frac{\delta^2 Q}{\delta A(\omega)\, \delta A(\omega')} \right)^{-1}_{A = A_\alpha} \right\rangle_I$$

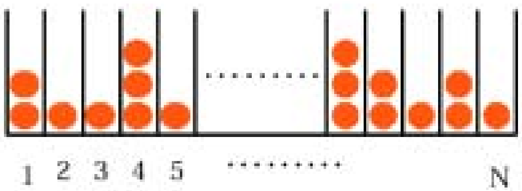


Shonon-Jaynes Entropy

- *Combinatorial construction (Monkey argument)*

Friedan ('72); Gull & Daniell ('79); Jaynes ('86); Skilling ('88)

Product of Poisson distribution → SJ entropy



$$P_\lambda(n) = \prod_{i=1}^{N} \frac{\lambda_i^{\,n_i} e^{-\lambda_i}}{n_i\,!}$$

$$\rightarrow \exp\left[\alpha \sum_{i=1}^{N} \left(A_i - m_i - A_i \ln\left(\frac{A_i}{m_i}\right)\right)\right]$$

$A_i = n_i / \alpha$

$m_i = \lambda_i / \alpha$

$n! \sim \exp[n \log n - n]$

- *Axiomatic construction*

Axom I : Locality

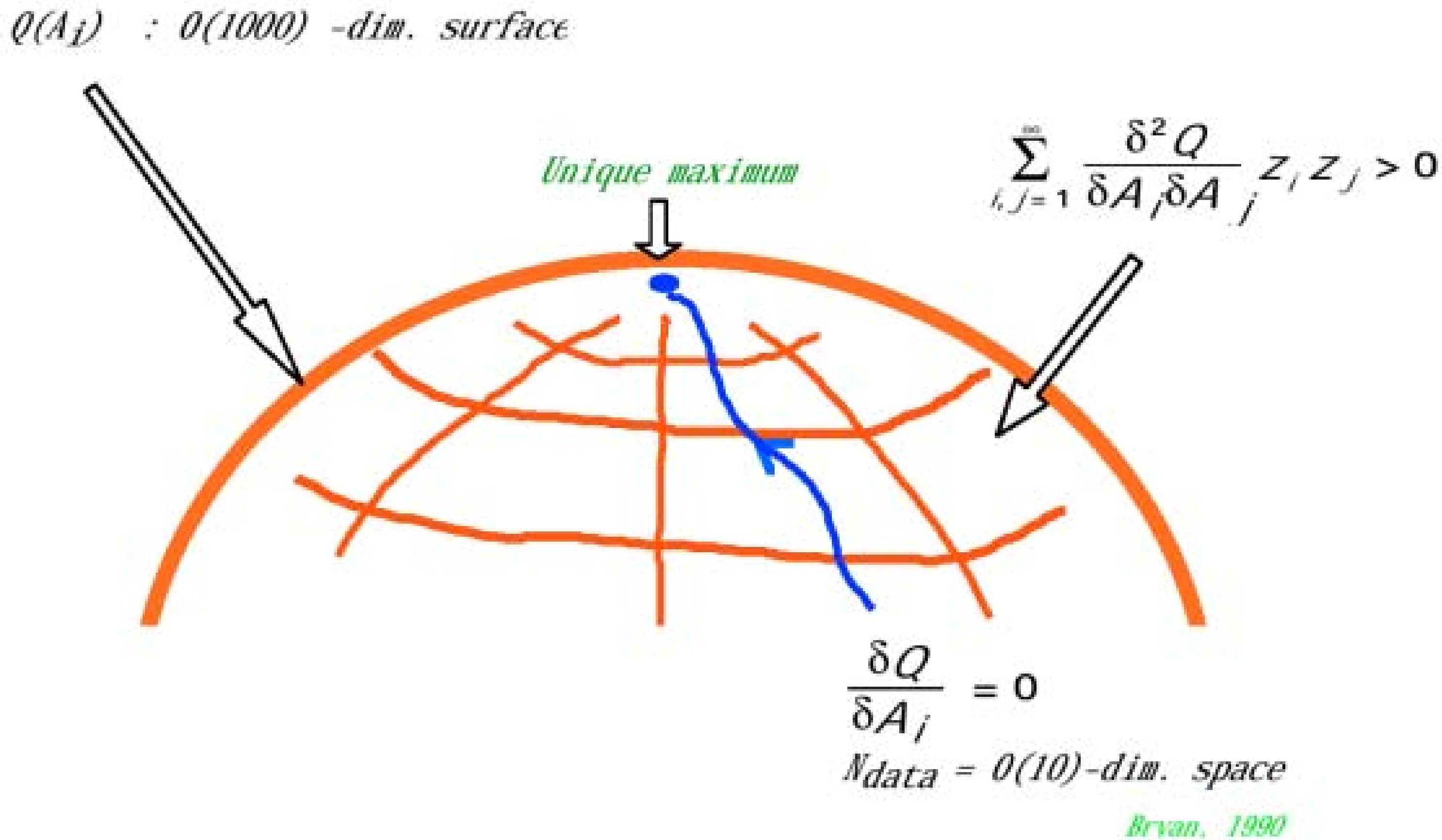
Axiom II : Coordinate invariance

Axiom III : System independence

Axiom IV : Scaling

Shanon (1948); Jaynes (1957), ······, Skilling ('88); Asakawa, T.H., & Nakahara, ('01)

Properties of hypersurface Q(A)



- *Kernel $K(\tau, \omega)$*

Free propagator with mass ω

$$D(\tau > 0) = \int_0^\infty K(\tau,\omega)\, A(\omega)\, d\omega$$

- *"continuum" kernel*

$$K_{cont}(\tau,\omega) = e^{-\omega\tau} = 2\omega \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \frac{e^{i\nu\tau}}{\omega^2 + \nu^2}\, d\nu$$

- *"lattice" kernel*

O(a) difference

$$K_{lat}(\tau,\omega) \equiv 2\omega \int_{-\pi/a}^{\pi/a} \frac{d\nu}{2\pi} \frac{e^{i\nu\tau}}{\omega^2 + \left(\frac{2}{a}\sin\frac{\nu a}{2}\right)^2}\, d\nu$$